An alternative image theory for indoor sound propagation

Byunghak Kong[a)]
School of Mechanical and Aerospace Engineering, Seoul National University
301-1214 College of Engineering, Seoul National University, Seoul, 151-742, Korea

Kyuho Lee[b)]
School of Mechanical and Aerospace Engineering, Seoul National University
311-105 College of Engineering, Seoul National University, Seoul, 151-742, Korea

Soogab Lee[c)]
Institute of Advanced Aerospace Technology and School of Mechanical and Aerospace Engineering, Seoul National University
311-105 College of Engineering, Seoul National University, Seoul, 151-742, Korea

An alternative image theory suitable for indoor sound propagation has been developed in the present study. This method predicted a sound pressure distribution over a closed space by adopting an image space concept for a locally reacting surface. A reflected sound field, which has been explained as a lot of the image source distributions, was illustrated by only several image spaces enclosing a real space along its boundaries. It was more convenient to simulate indoor sound propagation problems, especially including multiple reflecting phenomena, compared with the original image theory. The final result was obtained by folding the image spaces into the real space according to the symmetry. The numerical results obtained by the alternative method were compared with analytic solutions of the Helmholtz equation in one-dimensional space, and consequently, it is obviously confirmed that both results were in almost perfect agreement. A numerical simulation, which is a propagation of a broadband sound wave derived from a Gaussian pulse, was also conducted in three-dimensional space to verify the potential of the alternative approach.

1. INTRODUCTION

The sound field resulting from a source above the ground has been studied by many authors[1-4] because of the importance of this phenomenon. The most representative of them is the image theory. The image theory was originally derived by Lindell[5-6] to solve the Sommerfeld half-space problem, which simplifies the Sommerfeld integral using a vertical dipole in electric

[a)] email: bhgong03@snu.ac.kr
[b)] email: bukha16@snu.ac.kr
[c)] email: solee@snu.ac.kr

and magnetic fields. It is thought that the vertical dipole is separated into two different sources, in acoustics, a real source and an image source, with the ground of symmetry. That is, the sound field resulting from a point source above the boundary is interpreted as the superposition of those of the real and image sources by the image theory.

With this image formation, the integral is expressed as a simple equation because an image source is unique for a boundary. However, remarkable changes occur when this formulation is applied to not a half space but a closed space. The image source, unique over a half space, is determined according to a propagation path described through parameters such as the position of the real source as well as the boundaries of the closed space, the position of the receiver, and the number of reflections. In this case, the number of image sources sufficient to solve the problem increases drastically.

In this present study, an alternative image theory that is more effective to predict the sound field over a closed space than the original one was developed. An image space concept, which is an extended version of the image receiver, is adopted in developing this alternative method instead of the image source concept. It is another advantage that this method is more intuitive to be applied about broadband sound waves in time domain.

2. THE ALTERNATIVE METHOD

2.1 The image Receiver Concept

If the original image theory is used, the sound field over a half space generated by a point source is expressed by:

$$p = \frac{e^{ikR_1}}{R_1} + Q\frac{e^{ikR_2}}{R_2}, \tag{1}$$

where R is the distance from the real and image sources, and k is the wavenumber[3]. The spherical reflection coefficient, Q, is defined as a function of acoustical properties of the boundary, the normalized characteristic impedance, Z, and the incident angle, θ[3]. The formulations of these parameters are shown as Eqns. (2) and (3).

$$Q = R_p + \left(1 - R_p\right)F \tag{2}$$

$$R_p = \frac{cos\,\theta - 1/Z}{cos\,\theta + 1/Z} \tag{3}$$

If the boundary loss factor, F, is zero, the spherical reflection coefficient has the same value as the reflection coefficient for plane wave, R_p, in Eqns. (2). The first term in Eqns. (1) means the sound field induced by the real source. The second term indicates that of the image source. Similarly, the sound field over a closed space is expressed as follow:

$$p = \frac{e^{ikR_0}}{R_0} + Q_1\frac{e^{ikR_1}}{R_1} + Q_2\frac{e^{ikR_2}}{R_2} + Q_3\frac{e^{ikR_3}}{R_3} + Q_4\frac{e^{ikR_4}}{R_4} + \cdots. \tag{4}$$

As expressed in Eqns. (4), a number of image sources are required to estimate the sound field over a closed space by the original image theory. On the other hand, the alternative method is based on the image receiver concept so that no image source is needed. The image receiver concept is briefly described in Fig. 1. In this manner, the sound field at the image receiver induced by the real source means the reflected sound field, and corresponds the second term in Eqns. (1).

Though the image receiver concept is introduced, it remains unchanged that a number of the image receivers are required because they correspond to the propagation path like the image sources. Therefore the image space concept is proposed to settle this problem.

2.2 The Image Space

An image space is defined as a set of the image receivers about an arbitrary boundary. Therefore, it is more efficient to predict the acoustic pressure distribution over a closed space than the image source. The image spaces are simply described in two-dimensional space in Fig. 2. The order of each space implies the number of times the acoustic wave is reflected by the boundaries. That is, the first order space is composed of the image receivers where one reflection against the arbitrary boundary is considered, and, in the same way, the second one is composed of those where two reflections are considered.

The final result is obtained by folding the image spaces into the real one according to the symmetry. This procedure implies that the sound field over each space is summed up as in Eqns. (4). The schematic view of the folding concept over two-dimensional space is described in Fig. 3.

As the image space is introduced, it becomes easier to predict the sound pressure distribution over a closed space. In the following section, several numerical simulations are going to be shown to verify its validity and possibility.

3. NUMERICAL RESULTS

3.1 Pure tone

An analytic solution of the Helmholtz equation in one-dimensional space is expressed as follow when the right side is closed with the impedance wall.

$$p = e^{ikx} + Qe^{-ikx}. \tag{5}$$

On the other hand, if the alternative method is adopted, described as Fig. 4, and the sound field over the real space and image space is obtained by:

$$p_{real} = e^{ikx}, \tag{6}$$

$$p_{image} = Qe^{ikx}. \tag{7}$$

The impedance model used to define the reflection coefficient in Eqns. (3) is the Delany model[7], which is a widely used model for evaluating the acoustical properties of outdoor surfaces with a single parameter, the effective flow resistivity σ_e, to characterize the ground.

$$Z = 1 + 0.0571\left(\frac{f}{\sigma_e}\right)^{-0.754} - i0.087\left(\frac{f}{\sigma_e}\right)^{-0.732}. \tag{8}$$

The numerical computation is conducted with *f=500 Hz* and σ_e*=150 cgs rayls*, which is adequate for grass near an airport and a public building[8]. (*1 cgs rayls = 1,000 Pa s* m^{-2}) The results are shown in Fig. 5. The upper result in Fig. 5 is each signal over the real and image space, respectively. The amplitude and the phase of the signal over the image space are changed at the right boundary due to the reflection coefficient. The lower signals of Fig. 5 are the result of the comparison of the numerical and analytic solutions over the total and reflected field, respectively. As a result of this comparison, it is concluded that the alternative method can predict the sound field over a closed space effectively and precisely.

3.2 Broadband

For a broadband wave, a function is derived to describe the time history of acoustic pressure over three-dimensional space governed by the Helmholtz equation. This kind of function, Eqns. (9), is widely used to model a Gaussian pulse in physics.

$$p(r,t) = \frac{A}{r} exp\left[- ln\, 2 \left(\frac{r-t}{B}\right)^2\right] \tag{9}$$

The parameters, *A* and *B*, are defined as *5.0* and *0.05*, respectively. If all the boundaries are assumed a kind of perfectly reflecting surface, i.e. the reflection coefficient is unity, then the numerical results are described as Fig. 6. They are arranged in temporal sequence from upper left to lower right. With a time-varying function, the sound field over three-dimensional closed space is successively obtained.

4. CONCLUDING REMARKS

An alternative image theory which is adequate to simulate the sound field over a closed space is introduced in this study. The image space concept as an alternative to the image source concept is developed, and, consequently, it is possible to obtain the resulting solutions by folding the image spaces along the boundaries.

As the alternative method is implemented in an indoor sound propagation problem instead of the original method, it becomes possible to solve the problem in a simple and comprehensive way. If a boundary condition that is enough to represent the effect of reflection for broadband waves is introduced, the capability and the reliability of this method might be enlarged.

5. ACKNOWLEDGEMENTS

This work was supported by the New and Renewable Energy Program of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Knowledge Economy (No.20104010100490). And, this research was also supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No.20120002228).

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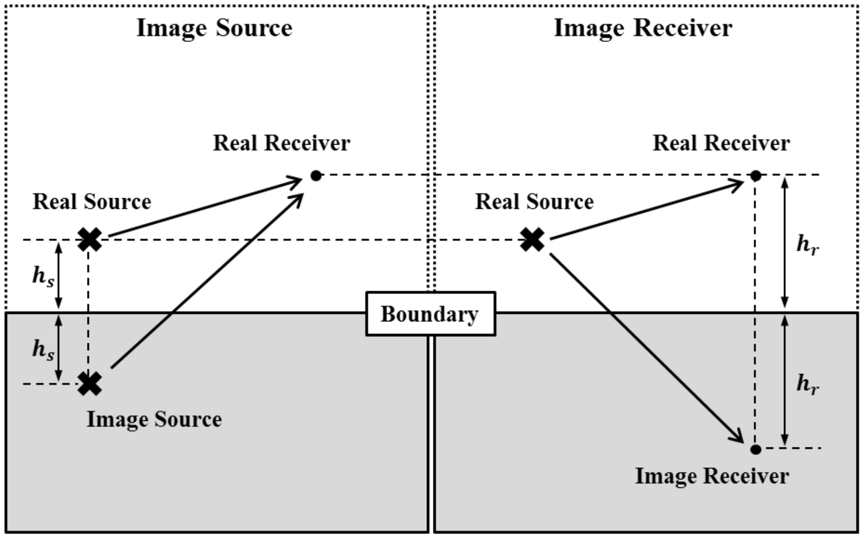

Fig. 1 - Comparison the image receiver concept with the image source concept.

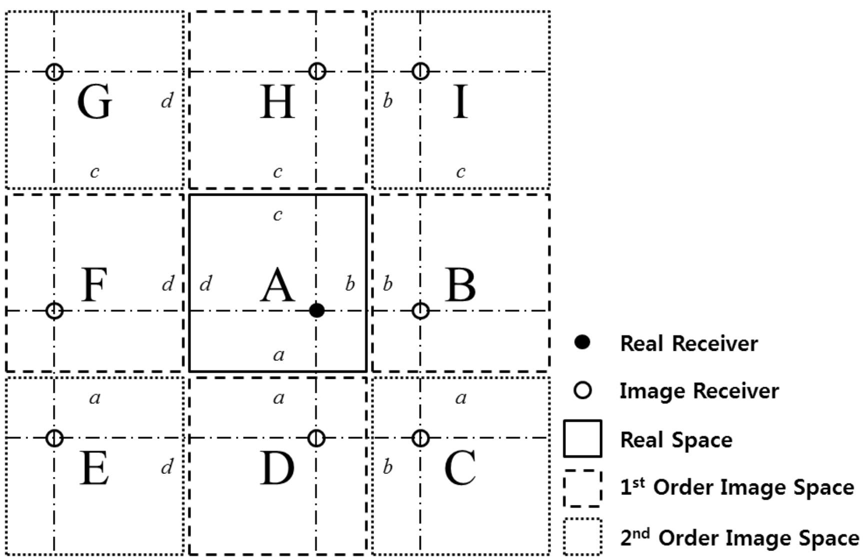

Fig. 2 - Description of the image space concept over two-dimensional space and the relations with its boundaries.

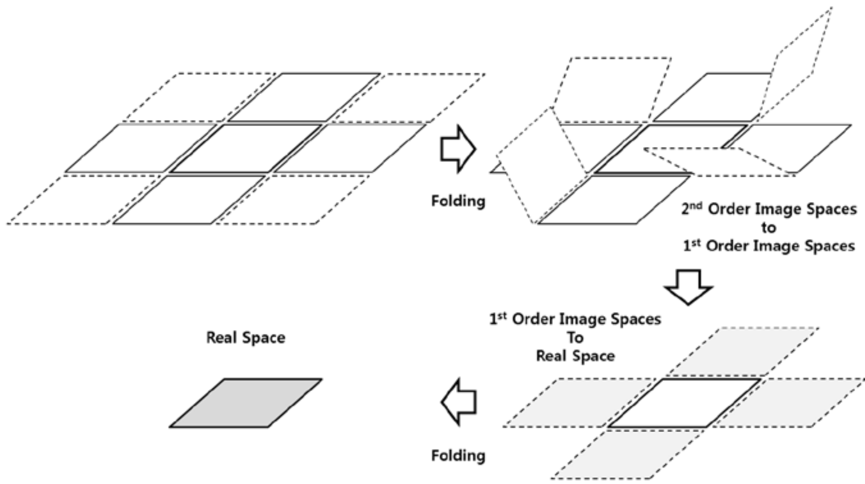

Fig. 3 - The schematic view of the folding concept over two-dimensional space.

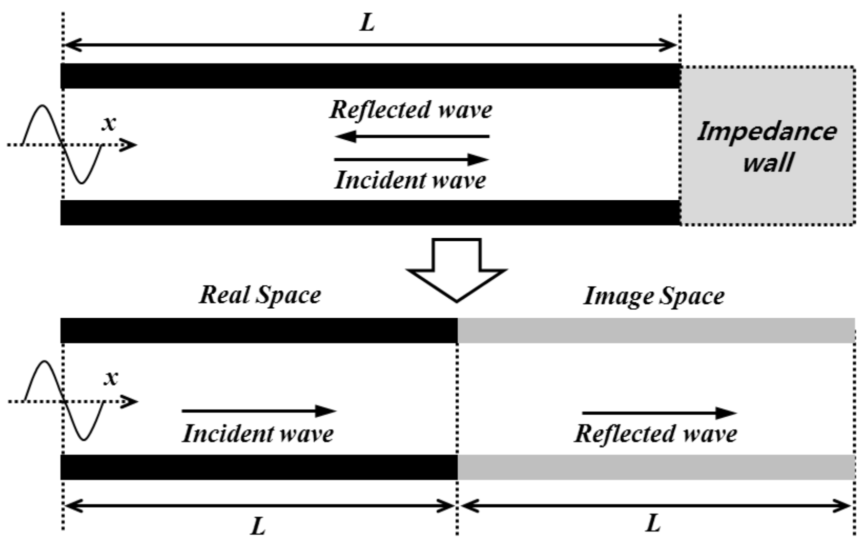

Fig. 4 - The schematics view of the alternative method over one-dimensional space with a closed end on the right side.

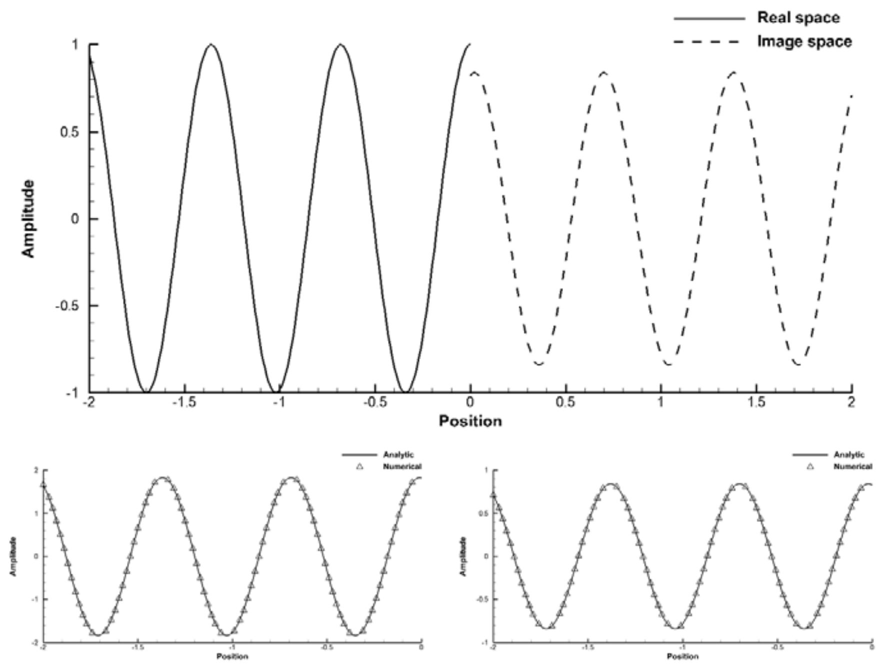

Fig. 5 - The numerical results over one-dimensional space for a pure tone wave.

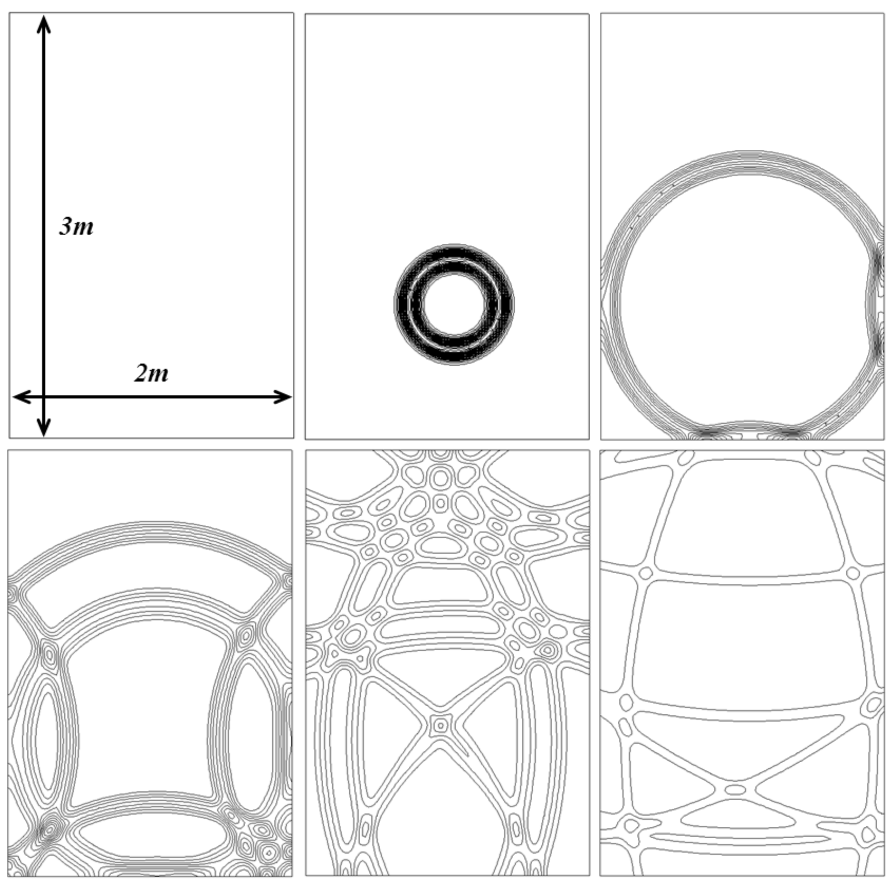

Fig. 6 - The numerical results over three-dimensional space for a broadband wave.